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**Online Homework**

**Focused Exercises for Math SAT**

**Skill Set 20: Defined Operations**

Many of the problems in this exercise set came from The College Board, writers of the SAT exam.

1. Let the operations  $\triangle$  and  $\square$  be defined for all real numbers  $a$  and  $b$  as follows:

$$a \triangle b = a + 3b$$

$$a \square b = a + 4b$$

If  $4 \triangle (5y) = (5y) \square 4$ , what is the value of  $y$ ?

2. Let the operation  $\hat{\cup}$  be defined by  $a \hat{\cup} b = \frac{a + b}{a - b}$  for all numbers  $a$  and  $b$ , where  $a \neq b$ . If  $1 \hat{\cup} 2 = 2 \hat{\cup} x$ , what is the value of  $x$ ?

- (A) 4
- (B) 3
- (C) 2
- (D) 1
- (E) 0

3. If  $k$ ,  $n$ , and  $r$  are integers, let  $k \blacklozenge (n, r)$  be defined to be true only if  $n < k < r$ . If  $-2 \blacklozenge (n, 0)$  is true, which of the following could be a possible value of  $n$ ?

- I.  $-3$
- II.  $-1$
- III.  $3$

- (A) I only
- (B) III only
- (C) I and II
- (D) I and III
- (E) II and III

4. For all numbers  $x$  and  $y$ , let the operation  $\square$  be defined by  $x \square y = xy - y$ . If  $a$  and  $b$  are positive integers, which of the following can be equal to zero?

- I.  $a \square b$
- II.  $(a + b) \square b$
- III.  $a \square (a + b)$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) I and III

5. Let  $\boxed{x}$  be defined as  $\boxed{x} = x^2 - x$  for all values of  $x$ . If  $\boxed{a} = \boxed{a-2}$ , what is the value of  $a$ ?

(A) 1

(B)  $\frac{1}{2}$

(C)  $\frac{3}{2}$

(D)  $\frac{6}{5}$

(E) 3

6. Let  $x \triangle_y z$  be defined as  $x \triangle_y z = x^y - z^y$  for all positive integers  $x$ ,  $y$ , and  $z$ . What is the value of  $10 \triangle_3 5$ ?

7. If  $k$  is a positive integer, let  $\boxed{k}$  be defined as the set of all multiples of  $k$ . All of the numbers in which of the following sets are also in all three of the sets  $\boxed{2}$ ,  $\boxed{3}$ , and  $\boxed{5}$ ?

(A)  $\boxed{5}$

(B)  $\boxed{6}$

(C)  $\boxed{10}$

(D)  $\boxed{21}$

(E)  $\boxed{60}$

8. If  $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$  is defined by  $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = ad + bc$ , what is the value of  $\begin{array}{|c|c|} \hline 2 & 3 \\ \hline 6 & 4 \\ \hline \end{array}$  ?

9. Let  $\begin{array}{|c|c|c|} \hline & b & \\ \hline a & & c \\ \hline & d & \\ \hline \end{array}$  be defined for all numbers  $a, b, c$  and  $d$  by

$\begin{array}{|c|c|c|} \hline & b & \\ \hline a & & c \\ \hline & d & \\ \hline \end{array} = ac - bd$ . If  $x = \begin{array}{|c|c|c|} \hline & 4 & \\ \hline 5 & & 2 \\ \hline & 1 & \\ \hline \end{array}$ , what is the value of  $\begin{array}{|c|c|c|} \hline & 10 & \\ \hline x & & 2 \\ \hline & 1 & \\ \hline \end{array}$  ?

- (A) 1
- (B) 2
- (C) 18
- (D) 38
- (E) 178

10, 11, 12.

Let  $\Delta n$  be defined for any positive integer  $n$  as the number obtained by writing the digits of  $n$  in reverse order, dropping any leading zeros that result.

For example,  $\Delta 5 = 5$ ,  $\Delta 30 = 3$ , and  $\Delta 123 = 321$ .

$$\Delta 45,000 - \Delta 43,000 =$$

- (A) 2
- (B) 20
- (C) 200
- (D) 2,000
- (E) 20,000

Which of the following is equal to  $\Delta 601 + \Delta 73$  ?

- (A)  $\Delta 53$
- (B)  $\Delta 134$
- (C)  $\Delta 143$
- (D)  $\Delta 341$
- (E)  $\Delta 638$

Which of the following must be true for all positive integers  $n$  ?

- I.  $\Delta(\Delta n) = n$
- II.  $\Delta(10 \cdot n) < 10 \cdot n$
- III.  $\Delta(1 + n) = 1 + \Delta n$

- (A) None
- (B) I only
- (C) II only
- (D) I and II only
- (E) I, II, and III

13., 14.

Refer to the following definitions for integers  $n$  greater than 1.

$$\triangle n = n^2 + n$$

$$\square n = n^2 - n$$

$$\triangle 5 - \square 4 =$$

- (A) 0
- (B) 8
- (C) 10
- (D) 18
- (E) 32

If  $m$  is an integer greater than 1, then  $\square m + 1 =$

- (A)  $\triangle m$
- (B)  $\triangle m + 1$
- (C)  $\triangle m - 1$
- (D)  $\square m + 1$
- (E)  $\square m - 1$

15. For all positive integers  $j$  and  $k$ , let  $j \square k$  be defined as the whole number remainder when  $j$  is divided by  $k$ . If  $13 \square k = 2$ , what is the value of  $k$ ?

16. Let  $\#$  be defined by  $z \# w = z^w$ . If  $x = 5 \# a$ ,  $y = 5 \# b$ , and  $a + b = 3$ , what is the value of  $xy$ ?

- (A) 15
- (B) 30
- (C) 75
- (D) 125
- (E) 243

17. For all positive integers  $n$ , let  $\boxed{n}$  equal the greatest prime number that is a divisor of  $n$ . What does  $\frac{\boxed{10}}{\boxed{12}}$  equal ?

18. For all integers  $x$ , let  $\boxed{x}$  be defined as follows:

$$\boxed{x} = \frac{x}{2} \text{ if } x \text{ is even.}$$

$$\boxed{x} = x^2 \text{ if } x \text{ is odd.}$$

If  $\boxed{2} + \boxed{3} = y$ , what is the value of  $y^3$  ?

19. For all numbers  $x$  and  $y$ , where  $x \neq y$ , let  $x \blacktriangle y$  be defined as  $\frac{x+y}{x-y}$ . If

$8 \blacktriangle w = \frac{4}{3}$ , what is the value of  $w$  ?

20. For  $x \geq -1$ ,  $\triangle x$  is defined by  $\triangle x = \sqrt{x+1}$ . Which of the following is equal to  $\triangle 15 - \triangle 8$  ?

(A)  $\triangle -1$

(B)  $\triangle 0$

(C)  $\triangle 1$

(D)  $\triangle 7$

(E)  $\triangle \sqrt{7}$

21. Let  $n \nabla$  be defined for all positive integer values of  $n$  as the sum of all positive even factors of  $2n$ . For example,  $3 \nabla = 2 + 6 = 8$ . What is the value of  $15 \nabla$  ?

22. For all positive integers  $a$  and  $b$ , if  $a \neq b$ , let  $a \diamond b$  be defined as  $\frac{a + b}{a - b}$ .  
What is the value of  $1,011 \diamond 11$  ?

- (A) 1.011  
(B) 1.022  
(C) 1.121  
(D) 2.111  
(E) 10.220
23.  $\lfloor x \rfloor$  is defined as the greatest integer less than  $x$ .  $\lceil x \rceil$  is defined as the least integer greater than  $x$ . What is the value of  $\lceil 25.8 \rceil - \lfloor 13.9 \rfloor$  ?