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Online Homework

Focused Exercises for Math SAT

Skill Set 20: Defined Operations

Many of the problems in this exercise set came from The College Board, writers of the SAT exam.

1. Let the operations \triangle and \square be defined for all real numbers a and b as follows:

$$a \triangle b = a + 3b$$

$$a \square b = a + 4b$$

If $4 \triangle (5y) = (5y) \square 4$, what is the value of y ?

2. Let the operation $\hat{\cup}$ be defined by $a \hat{\cup} b = \frac{a + b}{a - b}$ for all numbers a and b , where $a \neq b$. If $1 \hat{\cup} 2 = 2 \hat{\cup} x$, what is the value of x ?

- (A) 4
- (B) 3
- (C) 2
- (D) 1
- (E) 0

3. If k , n , and r are integers, let $k \blacklozenge (n, r)$ be defined to be true only if $n < k < r$. If $-2 \blacklozenge (n, 0)$ is true, which of the following could be a possible value of n ?

- I. -3
- II. -1
- III. 3

- (A) I only
- (B) III only
- (C) I and II
- (D) I and III
- (E) II and III

4. For all numbers x and y , let the operation \square be defined by $x \square y = xy - y$. If a and b are positive integers, which of the following can be equal to zero?

- I. $a \square b$
- II. $(a + b) \square b$
- III. $a \square (a + b)$

- (A) I only
- (B) II only
- (C) III only
- (D) I and II
- (E) I and III

5. Let \boxed{x} be defined as $\boxed{x} = x^2 - x$ for all values of x . If $\boxed{a} = \boxed{a-2}$, what is the value of a ?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{3}{2}$

(D) $\frac{6}{5}$

(E) 3

6. Let $x \triangle_y z$ be defined as $x \triangle_y z = x^y - z^y$ for all positive integers x , y , and z . What is the value of $10 \triangle_3 5$?

7. If k is a positive integer, let \boxed{k} be defined as the set of all multiples of k . All of the numbers in which of the following sets are also in all three of the sets $\boxed{2}$, $\boxed{3}$, and $\boxed{5}$?

(A) $\boxed{5}$

(B) $\boxed{6}$

(C) $\boxed{10}$

(D) $\boxed{21}$

(E) $\boxed{60}$

8. If $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$ is defined by $\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = ad + bc$, what is the value of $\begin{array}{|c|c|} \hline 2 & 3 \\ \hline 6 & 4 \\ \hline \end{array}$?

9. Let $\begin{array}{|c|c|c|} \hline & b & \\ \hline a & & c \\ \hline & d & \\ \hline \end{array}$ be defined for all numbers a, b, c and d by

$\begin{array}{|c|c|c|} \hline & b & \\ \hline a & & c \\ \hline & d & \\ \hline \end{array} = ac - bd$. If $x = \begin{array}{|c|c|c|} \hline & 4 & \\ \hline 5 & & 2 \\ \hline & 1 & \\ \hline \end{array}$, what is the value of $\begin{array}{|c|c|c|} \hline & 10 & \\ \hline x & & 2 \\ \hline & 1 & \\ \hline \end{array}$?

- (A) 1
- (B) 2
- (C) 18
- (D) 38
- (E) 178

10, 11, 12.

Let Δn be defined for any positive integer n as the number obtained by writing the digits of n in reverse order, dropping any leading zeros that result.

For example, $\Delta 5 = 5$, $\Delta 30 = 3$, and $\Delta 123 = 321$.

$$\Delta 45,000 - \Delta 43,000 =$$

- (A) 2
- (B) 20
- (C) 200
- (D) 2,000
- (E) 20,000

Which of the following is equal to $\Delta 601 + \Delta 73$?

- (A) $\Delta 53$
- (B) $\Delta 134$
- (C) $\Delta 143$
- (D) $\Delta 341$
- (E) $\Delta 638$

Which of the following must be true for all positive integers n ?

- I. $\Delta(\Delta n) = n$
- II. $\Delta(10 \cdot n) < 10 \cdot n$
- III. $\Delta(1 + n) = 1 + \Delta n$

- (A) None
- (B) I only
- (C) II only
- (D) I and II only
- (E) I, II, and III

13., 14.

Refer to the following definitions for integers n greater than 1.

$$\triangle n = n^2 + n$$

$$\square n = n^2 - n$$

$$\triangle 5 - \square 4 =$$

- (A) 0
- (B) 8
- (C) 10
- (D) 18
- (E) 32

If m is an integer greater than 1, then $\square m + 1 =$

- (A) $\triangle m$
- (B) $\triangle m + 1$
- (C) $\triangle m - 1$
- (D) $\square m + 1$
- (E) $\square m - 1$

15. For all positive integers j and k , let $j \square k$ be defined as the whole number remainder when j is divided by k . If $13 \square k = 2$, what is the value of k ?

16. Let $\#$ be defined by $z \# w = z^w$. If $x = 5 \# a$, $y = 5 \# b$, and $a + b = 3$, what is the value of xy ?

- (A) 15
- (B) 30
- (C) 75
- (D) 125
- (E) 243

17. For all positive integers n , let \boxed{n} equal the greatest prime number that is a divisor of n . What does $\frac{\boxed{10}}{\boxed{12}}$ equal ?

18. For all integers x , let \boxed{x} be defined as follows:

$$\boxed{x} = \frac{x}{2} \text{ if } x \text{ is even.}$$

$$\boxed{x} = x^2 \text{ if } x \text{ is odd.}$$

If $\boxed{2} + \boxed{3} = y$, what is the value of y^3 ?

19. For all numbers x and y , where $x \neq y$, let $x \blacktriangle y$ be defined as $\frac{x+y}{x-y}$. If

$8 \blacktriangle w = \frac{4}{3}$, what is the value of w ?

20. For $x \geq -1$, $\triangle x$ is defined by $\triangle x = \sqrt{x+1}$. Which of the following is equal to $\triangle 15 - \triangle 8$?

(A) $\triangle -1$

(B) $\triangle 0$

(C) $\triangle 1$

(D) $\triangle 7$

(E) $\triangle \sqrt{7}$

21. Let $n \nabla$ be defined for all positive integer values of n as the sum of all positive even factors of $2n$. For example, $3 \nabla = 2 + 6 = 8$. What is the value of 15∇ ?

22. For all positive integers a and b , if $a \neq b$, let $a \diamond b$ be defined as $\frac{a + b}{a - b}$.
What is the value of $1,011 \diamond 11$?

- (A) 1.011
(B) 1.022
(C) 1.121
(D) 2.111
(E) 10.220
23. $\lfloor x \rfloor$ is defined as the greatest integer less than x . $\lceil x \rceil$ is defined as the least integer greater than x . What is the value of $\lceil 25.8 \rceil - \lfloor 13.9 \rfloor$?