1. Compute the following indefinite and definite integrals. Make sure that any substitutions you use are clearly worked out. State clearly which trigonometric identities you used, if any.

a) $\int (\sin x)^{\frac{2}{3}} \cos^5 x \, dx$

b) $\int_1^{\sqrt{2}} \frac{x}{\sqrt{4-x^2}} \, dx$

c) $\int \frac{1}{(t+5)\ln(t+5)} \, dt$

d) $\int e^{4x}\sqrt{1+e^{4x}} \, dx$

Consider the function $f(x) = \frac{1}{x+1}$ over the interval $(0,1)$.

(a) Sketch $f(x)$ over the given interval, and calculate upper and lower bounds for the area under the curve between $x = 0$ and $x = 1$ by considering a single rectangle for each bound.

(b) Evaluate the following integral exactly

$$\int_0^1 \frac{1}{x+1} \, dx.$$

Also calculate a decimal approximation, to two decimal places.
Is your answer between the bounds calculated above?

(c) Explain how the Mean Value Theorem assures us that there is a $c \in (0,1)$ such that $f'(c) = -\frac{1}{2}$. You don't need to find a value for c.

2. Vectors $\mathbf{a} = (-2,-1,0)$, $\mathbf{b} = (3,1,-2)$, $\mathbf{c} = (1,2,2)$ are given .

(a) Compute $(\mathbf{c} - \mathbf{a}) \cdot \mathbf{b}$.

(b) Calculate the cosine of the angle between $\mathbf{a}$ and $\mathbf{c}$.

(c) Determine the orthogonal projection of $\mathbf{a}$ on $\mathbf{c}$.

(d) Determine which, if any, of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are perpendicular to each other .

(e) Compute $\mathbf{a} \times \mathbf{b}$.

(f) Find the volume of the parallelepiped with four of its vertices at $(0,0,0)$, $(1,1,2)$, $(1,1,-1)$ and $(3,-1,2)$.
(The other four vertices are at $(2,2,1)$, $(4,0,1)$, $(4,0,4)$ and $(5,1,3)$) .

3. For each of the following limits, evaluate it or show that it does not exist. You may use L'Hopital's Rule if it applies. Justify your answers.

(a) $\lim_{t \to 2} \frac{t+3}{t^2-2}$.

(b) $\lim_{h \to 0} \frac{1}{h} \left(\frac{1}{\sqrt{4+h}} - \frac{1}{2} \right)$.

c). $\lim_{x \to 2} \frac{x^2-4}{\sqrt{2}-\sqrt{x}}$

d) $\lim_{\theta \to 0} \frac{\sin^2 \theta}{1-\cos\theta}$.

e) $\lim_{t \to 0} e^t(1-e^t)^5$.

4. a) Use implicit differentiation to obtain an expression for $\frac{dy}{dx}$ in terms of x and y, given that

$$x^2 y + e^{\sin(x)+y} = 1$$

b) If $\frac{dy}{dt} = e - 1$ when $x = 0$ and $y = 1$, find $\frac{dy}{dt}$.

6. An open-ended right cone is required to have a volume of $\frac{\pi}{3} m^3$ (supposing we could measure that volume). Find the height and radius that minimise the cone's surface area (ignoring the circular base).

Hint: The volume of the cone is. $V(r,h) = \frac{1}{3}\pi r^2 h$ Surface area is $S(r,h) = \pi r\sqrt{r^2+h^2}$.

7. The bottom of a $10-foot$ ladder is going away from the wall at $\frac{dx}{dt} = 2$ feet per second.

   a) How fast is the top going down the wall?

   b) Draw the right triangle to find $\frac{dx}{dt}$ when the height y is $6feet$.

8. Consider Find $\frac{dy}{dt} = 3(y-1)^2$

   a) Find y(t) given that $y(0) = 2$ as the initial condition. (Hint: Solve the differential equation by separating the variables.)

b) Use your solution to find the time t at which $y = 0$.