First system

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

Controllability:

$$rank\begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix} =$$

$$rank\begin{bmatrix} 1 & 5 & 30 & 70 & 610 & 1506 & 12920 & 31800 \\ 2 & 6 & 70 & 174 & 1490 & 3666 & 31480 & 77496 \\ 3 & 7 & 40 & 96 & 830 & 2046 & 17560 & 43224 \\ 4 & 8 & 80 & 200 & 1710 & 4206 & 36120 & 88920 \end{bmatrix} = 3$$

Since n=4 and rank=3 the system is uncontrollable.

Observability:

$$rank\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} =$$

$$rank\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 41 & 51 & 61 & 71 \\ 97 & 123 & 149 & 175 \\ 844 & 1068 & 1292 & 1516 \\ 2060 & 2604 & 3148 & 3692 \\ 17864 & 22584 & 27304 & 32024 \\ 43528 & 55032 & 66536 & 78040 \end{bmatrix} = 2$$

Since n=4 and rank=2 system is unobservable.

First step would be the Controllable/Uncontrollable Decomposition.

Our transformation matrix consists of first three columns of controllability matrix, since they are linearly independent. We have to add one more (arbitrarily chosen) linearly independent column so that T is nonsingular.

$$T = \begin{bmatrix} 1 & 5 & 30 & 0 \\ 2 & 6 & 70 & 0 \\ 3 & 7 & 40 & 0 \\ 4 & 8 & 80 & 1 \end{bmatrix}, \; rank(T) = 4$$