Suppose that the unsteady temperature distribution has to be obtained on a one-dimensional wall problem. See the following figure

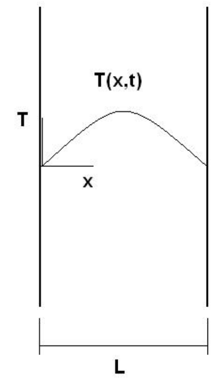

Figure 1: Heated wall suddenly exposed to a lower temperature at the boundaries

The governing equation is

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

Introducing $x' = x/L$, $T' = T/T_0$ and $t' = \alpha t/L^2$, then

$$\frac{\partial T'}{\partial t'} = \frac{\partial^2 T'}{\partial x'^2} \tag{2}$$

Dropping the prime, we have

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \tag{3}$$

The initial condition is

$$T = 1 \ for \ 0 \leq x \leq 1 \tag{4}$$

The boundary conditions are (for $t > 0$):

$$T = 0 \ for \ x = 0 \ and \ x = 1 \tag{5}$$

The analytical solution to the problem is

$$T = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} sin\{(2n-1)\pi x\} \cdot exp\{-(2n-1)^2 \pi^2 t\} \tag{6}$$

Using the Finite Volume Method (FVM) along with the Tridiagonal Matrix Algorithm (TDMA), this problem can be solved easily and efficiently.

- Using the finite volume method with the TDMA solution algorithm, write a code to solve the transient one-dimensional heat-conduction problem described above. Use your code to solve the problem for the conditions given below, and compare to the analytical solution. Describe and explain any differences between the solutions, and any differences from the analytical solution.

  - a) Use 21 volumes and the Crank-Nicolson method ($f = 0.5$) using the following time steps: 0.0005, 0.001, 0.002.
  - c) Use the Crank-Nicolson method ($f = 0.5$) and a time step of 0.001 using the following number of volumes: 11, 21, 101.
  - b) Use 21 volumes and a time step of 0.001, for the explicit ($f = 0$), the Crank-Nicolson ($f = 0.5$) and the implicit ($f = 1$) solution methods.

  For each part, examine the temperature distributions across the plate for times of 0.008, 0.2, 0.4, and 1.0. What combination of relaxation, time-step, and grid density is optimal for this simulation. There is no "correct" answer, but I would like you to provide a convincing argument for your selection (*This is the most important part of your report!*).

- Notes:

  - Be aware of the difference in the discretized equations at the boundaries.
  - Write your code in such a way that the number of nodes and the time step are variable, to ease its use for the solutions.
  - Although the general code can be written such that the TDMA algorithm is used for all values of f, the explicit method ($f = 0$) does not require the TDMA algorithm and can be coded separately if you desire. However, try the general method first.
  - Use appropriate line/symbol graphs to compare data for this analysis. Do not show any contour plots.