1. Find the limits (a) $\lim_{x\to 0^-} 2x$, (b) $\lim_{x\to 3^+} x$, (c) $\lim_{x\to 3^+} 5$, (d) $\lim_{x\to -1^-} |x+2|$? In each case if you use a limit law, mention which one you are using.
2. Sketch by hand the graph of the function

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } x < -1 \\ x & \text{if } x \geq -1 \end{cases}$$

   Find $\lim_{x\to -1^-} f(x)$. Find $\lim_{x\to -1^+} f(x)$. Are these two limits the same? What can you say about $\lim_{x\to -1} f(x)$?
3. Use the Limit Laws to find the limit. *Graph each function before you submit the solution to check if the graph supports your conclusion. No need to include the graph though. You could use the grapher linked from the top right corner of this page.* $\lim_{x\to 3}(x^2 - x + 2)$.
4. Use the Limit Laws to find the limit $\lim_{x\to 3}(x^2 - x + 2/x^3)$. [Note: $x^2 - x + 2/x^3 \neq \frac{x^2-x+2}{x^3}$.
5. Use the Limit Laws to find the limit $\lim_{x\to -3} \frac{x^2-9}{x+3}$.
6. Use the Limit Laws to find the limit $\lim_{x\to 7} \frac{\sqrt{x}-\sqrt{7}}{x-7}$
7. Use the Limit Laws to find the limit $\lim_{x\to 5} \frac{x^3-125}{x-5}$
8. Does the limit $\lim_{x\to 1} \frac{x^2-x-1}{x-1}$ exist?
9. Use Limit Laws to find the limit $\lim_{x\to 1} \frac{x^2+x-2}{x-1}$.
10. Graph the function $f(x) = \begin{cases} \sin\frac{\pi}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ where $-0.2 \leq x \leq 0.2$. Then graph $f(x)$ where x varies over a smaller interval, for example, $-.06 \leq x \leq 0.06$. Determine $\lim_{x\to 0} \sin\frac{\pi}{x}$ if it exists. Describe in a paragraph the behavior of the values of $f(x)$ near $x = 0$. Include graphs in your solutions.
11. Graph the function

$$f(x) = \begin{cases} x^2 \sin\frac{\pi}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \quad \text{where } -0.2 \leq x \leq 0.2.$$

   Then graph $f(x)$ where x varies over a smaller interval, for example, $-.08 \leq x \leq 0.08$. Describe in a paragraph the behavior of the values of $f(x)$ near $x = 0$. What are the two functions $g(x)$ and $h(x)$ squeezing $f(x)$, that is such that $h(x) \leq f(x) \leq g(x)$? What is $\lim_{x\to 0} h(x)$?

example:
wrong :
$\lim_{x\to 2}(x^2 + x)$
$= x^2 + x = 2^2 + 2 = 6.$
correct :
$\lim_{x\to 2}(x^2 + x)$
$= \lim_{x\to 2}(x^2) + \lim_{x\to 2} x$
$= (\lim_{x\to 2} x)^2 + \lim_{x\to 2} x$
$= 2^2 + 2 = 6.$

and what is $\lim_{x\to 0} g(x)$? Determine $\lim_{x\to 0} x^2 \sin\frac{\pi}{x}$ if it exists. Include graphs in your solutions.

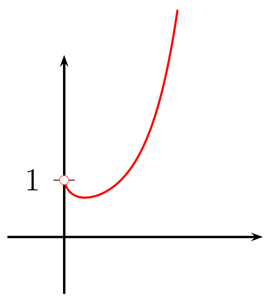
12. Discuss on the forum but do not submit. Let $f(x)$ be the smallest digit that appears in the decimal expansion of x. For example, $f(0.123123123123\ldots) = 0$ and $f(1.1111\ldots) = 1$, and $f(2.10101010101\ldots) = 0$. Find if they exist these limits: (a) $\lim_{x\to 1} f(x)$, (b) $\lim_{x\to 2} f(x)$?
13. Discuss on the forum but do not submit. We define f on a colored interval $[0, 1)$.

$$f(x) = \begin{cases} x \text{ if } x \text{ is blue} \\ 1 \text{ otherwise.} \end{cases}$$

(a) Determine $\lim_{x\to 1^-} f(x)$ in the case when the entire interval $[0, 1)$ is blue? (b) Determine $\lim_{x\to 1^-} f(x)$ in the case when the entire interval $[0, 1)$ is black? (c) Does there exist $\lim_{x\to 1^-} f(x)$ if near 1 (in every neighborhood of 1) there are numbers colored blue and there are numbers colored black? Hand sketch a possible graph of f.

Have a sheet of paper and a pen. This quiz is required and it is part of homework but no need to explain your choice.

1. Based on this graph of $y = x^x$,



(A) $\lim_{x\to 0+} x^x = 1$
(B) $\lim_{x\to 0-} x^x = 1$
(C) $\lim_{x\to 0} x^x$ is a number slightly less than 1.
(D) $\lim_{x\to 0-} x^x = 2$.

2. $\lim_{x\to 2-} \frac{x^3-8}{x-2}$
(A) 12
(B) -2
(C) exists but it is equal to a number not listed here
(D) the limit does not exist since the denominator is 0 when $x = 2$.

3. As $x \to 2^-$, the values of $f(x)$ approach -3. However, as $x \to 2+$ the values of $f(x)$ do not approach any particular number. Which of the statements is then correct?
(A) $\lim_{x\to 2-} f(x) = -3$ but neither $\lim_{x\to 2+} f(x)$ nor $\lim_{x\to 2} f(x)$ exists.
(B) $\lim_{x\to 2+} f(x) = -3$
(C) $\lim_{x\to 2} f(x) = -3$
(D) All above are incorrect.

4. Which of the following equations is *correct*?
(A) $\lim_{h\to 0} \frac{h}{h^2} = \frac{\lim_{h\to 0} h}{\lim_{h\to 0} h^2} = \frac{0}{0^2} = 0$.
(B) $\lim_{h\to 0} \frac{h^2}{h} = \frac{\lim_{h\to 0} h^2}{\lim_{h\to 0} h} = \frac{0^2}{0} = 0$.
(C) $\lim_{h\to 0} \frac{h^2}{h} = \lim_{h\to 0} h = 0$.
(D) Since the denominator goes to zero, all the above equations are not correct.

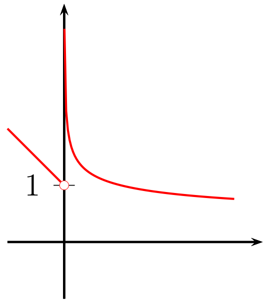
5. Let

$$f(x) = \begin{cases} \frac{1}{2}x & \text{if } x < -1 \\ 2x & \text{if } x \geq -1 \end{cases}$$

(A) $\lim_{x\to -1^-} f(x) = -\frac{1}{2}$ and $\lim_{x\to -1^+} f(x) = -2$
(B) $\lim_{x\to -1^-} f(x) = -2$ and $\lim_{x\to -1^+} f(x) = -\frac{1}{2}$
(C) $\lim_{x\to -1^-} f(x)$ does not exist and $\lim_{x\to -1^+} f(x)$ does not exist
(D) $\lim_{x\to -1^-} f(x) = 1$ and $\lim_{x\to -1^+} f(x) = 1$

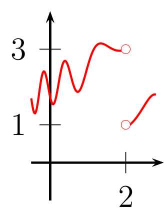
6. **(A)** $\lim_{x\to 1} \frac{x^2-x+1}{x+1} = \frac{1}{2}$
(B) $\lim_{x\to 1} \frac{x^2-x+1}{x+1}$ does not exist
(C) $\lim_{x\to 1} \frac{x^2-x+1}{x+1}$ cannot be calculated since $x^2 - x + 1$ does not factor
(D) none of the aboove.

7. Based on this graph of $y = f(x)$ we conclude that



(A) $\lim_{x\to 0} f(x) = 1$
(B) $\lim_{x\to 0} f(x) = 0$
(C) $\lim_{x\to 0} f(x)$ does not exist
(D) $\lim_{x\to 0} f(x) = 0$ or $\lim_{x\to 0} f(x) = 1$.

8. Based on this graph of $y = f(x)$ we conclude that



(A) $\lim_{x\to 2^-} f(x) = 3$
(B) $\lim_{x\to 2^-} f(x) = 2$
(C) $\lim_{x\to 2^-} f(x)$ does not exist
(D) $\lim_{x\to 2^-} f(x) = 1$.